

Modeling Timing Uncertainty in Cislunar Space using GPS Time Transfer

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I. Motivation

Planned robotic and human presence in cislunar space is creating a need for reliable, scalable navigation solutions and timekeeping. As such, several organizations (NASA and the ESA included [1,2]) have made plans for the establishment of lunar navigation satellite systems (LNSS). Earth GNSS utilizes ground-based stations to track and correct satellite clocks; in cislunar space, this is complicated by distance and more challenging communications. The reception of weak GNSS signals and use for time transfer is already being explored for lunar distances [3]; the work presented here investigates the effect and sensitivities of clock drift between navigation solutions on end-user navigation performance. Such techniques can also be utilized on standalone missions requiring high timing accuracy.

II. Clock Selection and Modeling

Many spaceborne oscillators can be modelled as random walk- and run-type process [4]:

$$\mathbf{x}(t) = \Phi(t, t_0)\mathbf{x}(t_0) + G$$

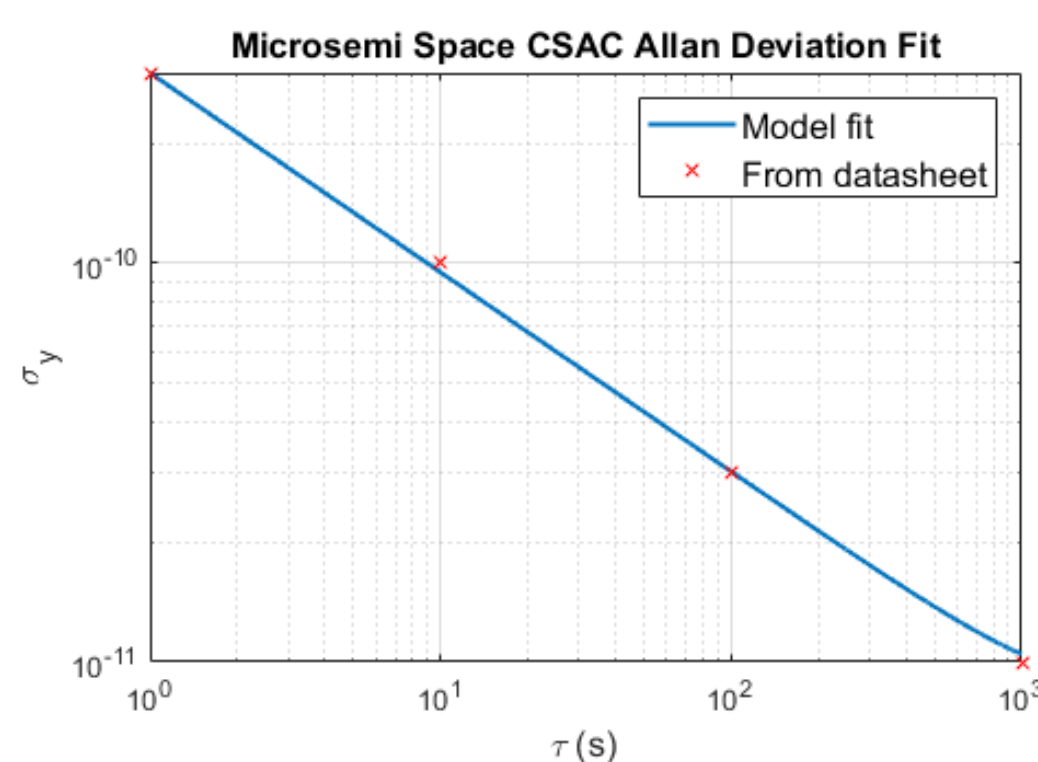
$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}, \quad \Phi(t, t_0) = \begin{bmatrix} 1 & t - t_0 & (t - t_0)^2/2 \\ 0 & 1 & t - t_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G \sim N(0, Q), \quad Q = \begin{bmatrix} \sigma_1^2 t + \sigma_2^2 \frac{t^3}{3} + \sigma_3^2 \frac{t^5}{20} & \sigma_2^2 \frac{t^2}{2} + \sigma_3^2 \frac{t^4}{8} & \sigma_3^2 \frac{t^3}{6} \\ \sigma_2^2 \frac{t^2}{2} + \sigma_3^2 \frac{t^4}{8} & \sigma_2^2 t + \sigma_3^2 \frac{t^3}{3} & \sigma_3^2 \frac{t^2}{2} \\ \sigma_3^2 \frac{t^3}{6} & \sigma_3^2 \frac{t^2}{2} & \sigma_3^2 t \end{bmatrix}$$

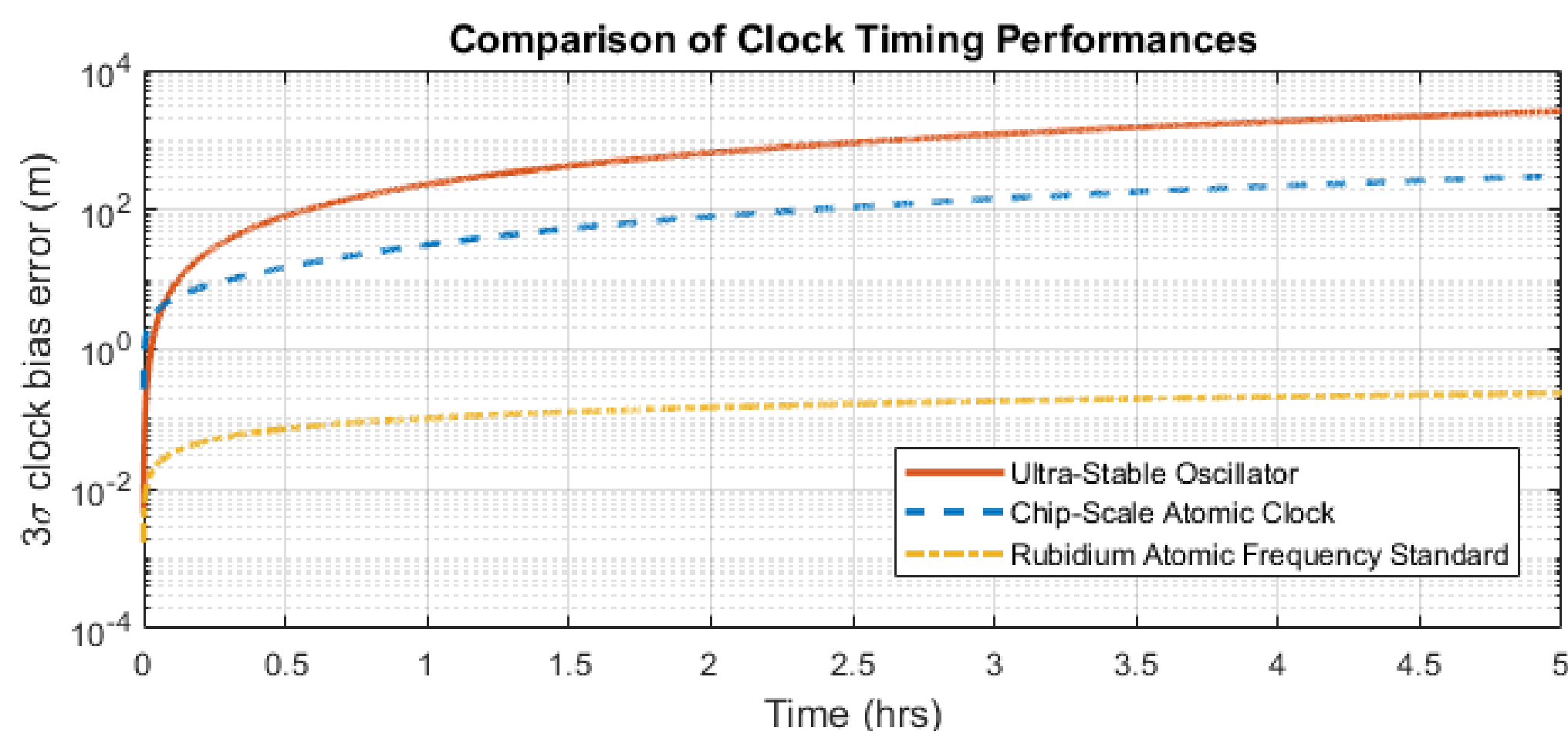
σ_1 , σ_2 , and σ_3 can be derived from Allan or Hadamard variance models using the relations below:

$$\sigma_y^2(\tau) = \frac{\sigma_1^2}{\tau} + \frac{\sigma_2^2 \tau}{3} + \frac{\tau^2}{2} c_3^2$$

$$\sigma_H^2(\tau) = \frac{\sigma_1^2}{\tau} + \frac{\sigma_2^2 \tau}{6} + \frac{11}{120} \sigma_3^2 \tau^3 + \frac{\mu_3^2 \tau^4}{6}$$



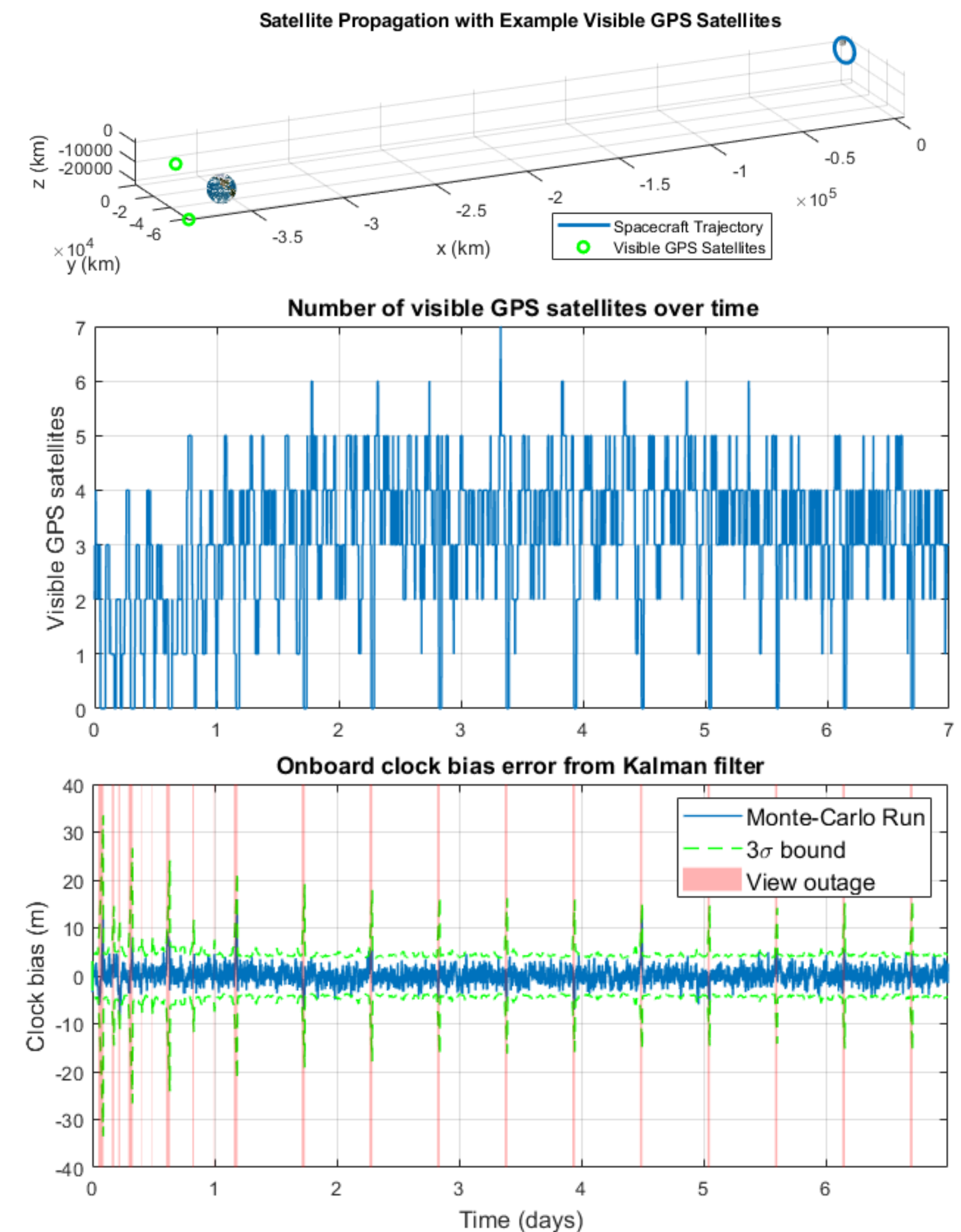
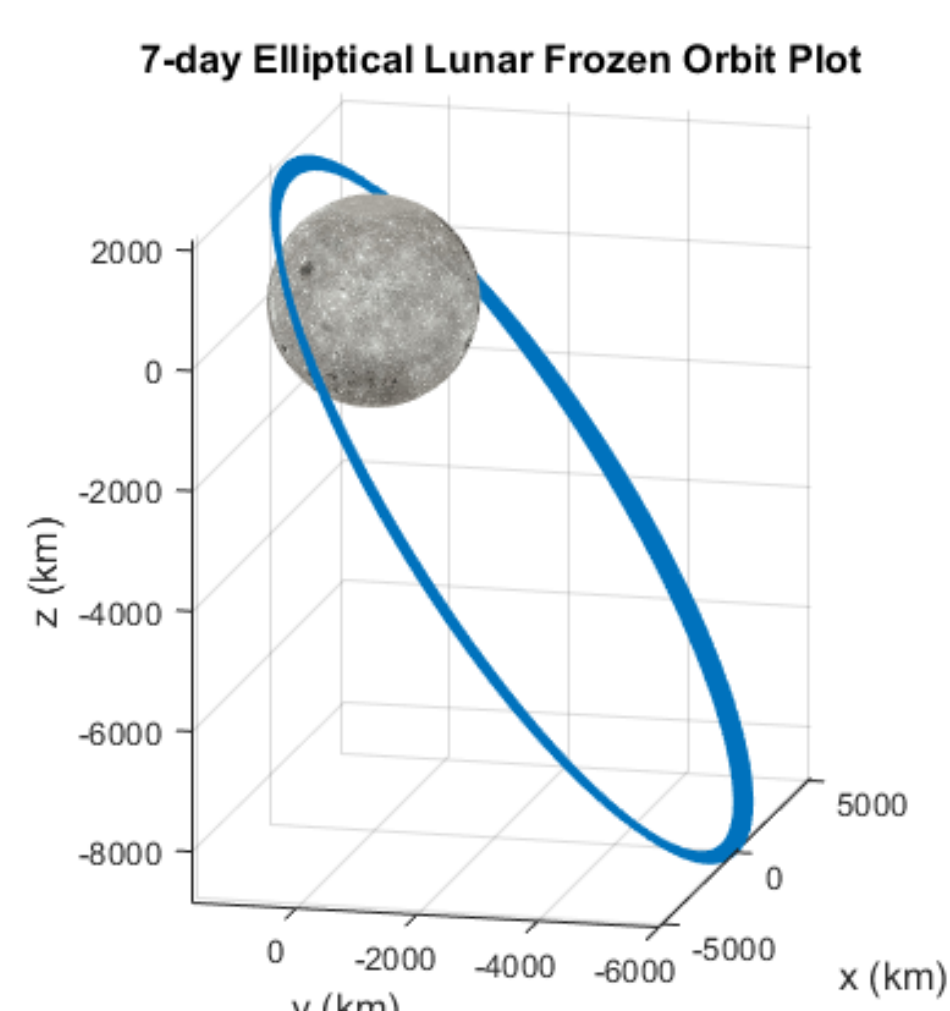
Different oscillator types offer varying levels of performance, price, and size / weight / power (SWaP). A key metric is the drift of phase bias over time, as it directly correlates to measurement uncertainty in lunar navigation systems. Below compares various timing standards in terms of this bias.



III. Time Transfer

Setup

- Kalman filter with previously described dynamics
- Pseudorange and pseudorange-rate residuals are measurements (from code and carrier tracking, respectively)
- Advanced GPS receiver
 - 14.8 m and 0.009 m/s 3σ UERE/UERRE uncertainty [5]
 - 40° maximum off-boresight angle link can close at moon [6]
- Satellite OD uncertainty of 10m and 0.01 m/s 3σ
- Clock is a Microsemi Space CSAC with listed performance [7]
- 7-day simulation, measurements 1/min
- Ignore signals passing through atmosphere (up to 1000 km in altitude)



Example 7-day GPS time transfer simulation for ELFO satellite, starting at 27 Oct 2023 00:00:00

IV. Results

Parameter	Change	3 σ Bound of MC Run ¹ (m)	Median 3 σ Bound of Filter (m)	3 σ Bound of MC Run ¹ (mm/s)	Median 3 σ Bound of Filter (mm/s)
	Nominal	5.309	4.392 (99.8%) ²	3.131	2.754 (99.8%) ²
OD Error	x10	10.895	10.800 (99.4%) ²	3.746	3.406 (99.7%) ²
	x0.1	4.648	3.975 (99.8%) ²	3.125	2.744(99.6%) ²
Clock Error	x10	26.901	9.423 (99.7%) ²	17.690	5.103 (99.9%) ²
	x0.1	1.937	1.788 (99.7%) ²	0.854	0.867 (99.8%) ²
Meas. Rate	x10	3.511	2.573 (99.8%) ²	2.481	1.641 (99.8%) ²
	x0.1	7.698	7.063 (99.8%) ²	4.708	4.081 (99.9%) ²
Est. vs True OD Error	10:1 ³	8.195	10.800 (100%) ²	3.365	3.406 (99.9%) ²
	0.1:1 ³	5.322	3.975 (99.5%) ²	3.280	2.744 (99.8%) ²
Est. vs True Clock Error	10:1 ³	9.645	9.423 (99.9%) ²	5.527	5.103 (99.9%) ²
	0.1:1 ³	10.694	1.788 (42.3%) ²	7.120	0.867 (30.9%) ²
Off Bore-sight Angle	+10°	4.561	3.833 (99.6%) ²	2.532	2.377 (99.8%) ²
	-10°	8.405	5.341 (99.7%) ²	4.706	3.315 (99.6%) ²

Key: Greatly improved, Greatly worsened, Relatively unchanged

¹ 10 Monte-Carlo runs per line, varying clock state and measurement noise each run
² Percent of data points within 3 σ bounds (should be ~ 99.7%)
³ Ratio of assumed error covariance used in filter to true error covariance (true error unchanged)

A sensitivity analysis was conducted for the same dataset while varying simulation parameters; results are in the table above, with notable changes highlighted. Some key takeaways:

- Very sensitive to changes in clock; better oscillators greatly improve total accuracy and resiliency to measurement outages
- Improved OD accuracy yields diminishing returns; error dominated by GPS
- Underestimating process noise results in very poorly fit filter; the impacts of underestimating OD error are less pronounced

VI. References

- [1] Babu, N., "LunaNet Interoperability Specification Document," NASA, Sep 07 2022.
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- [3] Bhamidipati, S., Mina, T., Sanchez, A., and Gao, G., "A Lunar Navigation and Communication Satellite System with Earth-GPS Time Transfer: Design and Performance Considerations," presented at the 35th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2022), Denver, Colorado, 2022. <https://doi.org/10.33012/2022.18365>
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- [5] Moorefield, F., "Global Positioning System Standard Positioning Service Performance Standard," U.S. Department of Defense, April 2020.
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- [7] "Space CSAC," Datasheet SA.45s, Microchip, March 2019.